arXiv:cs/0702068v2 [cs.IT] 16 Apr 2007

DISTRIBUTED DECISION THROUGH SELF-SYNCHRONIZING SENSOR NETWORKS IN THE PRESENCE OF PROPAGATION DELAYS AND NONRECIPROCAL CHANNELS

Gesualdo Scutari, Sergio Barbarossa and Loreto Pescosolido

Dpt. INFOCOM, Univ. of Rome "La Sapienza ", Via Eudossiana 18, 00184 Rome, Italy

E-mail: {scutari, sergio, loreto}@infocom.uniroma1.it.

ABSTRACT

In this paper we propose and analyze a distributed algorithm for achieving globally optimal decisions, either estimation or detection, through a self-synchronization mechanism among linearly coupled integrators initialized with local measurements. We model the interaction among the nodes as a directed graph with weights dependent on the radio interface and we pose special attention to the effect of the propagation delays occurring in the exchange of data among sensors, as a function of the network geometry. We derive necessary and sufficient conditions for the proposed system to reach a consensus on globally optimal decision statistics. One of the major results proved in this work is that a consensus is achieved for any bounded delay condition if and only if the directed graph is quasi-strongly connected. We also provide a closed form expression for the global consensus, showing that the effect of delays is, in general, to introduce a bias in the final decision. The closed form expression is also useful to modify the consensus mechanism in order to get rid of the bias with minimum extra complexity.

1. INTRODUCTION AND MOTIVATIONS

Endowing a sensor network with self-organizing capabilities is undoubtedly a useful goal to increase the resilience of the network against node failures (or simply switches to sleep mode) and avoid potentially dangerous congestion conditions around the sink nodes. Decentralizing decisions decreases also the vulnerability of the network against damages to the sink or control nodes. Distributed computation over a network has a long history, starting with the pioneering work of Tsitsiklis, Bertsekas and Athans [1] on asynchronous agreement problem for discrete-time distributed decision-making systems and parallel computing [2]. A simple, yet significant, form of in-network distributed computing is achieving a consensus about one common observed phenomenon, without the presence of a fusion center. Linear average consensus techniques have received great attention in the recent years [3]−[5]. An excellent tutorial on distributed consensus techniques is given in [6].

Consensus may be also seen as a form of self-synchronization among coupled dynamical systems. In [7, 8], the authors showed how to use the self-synchronization capabilities of a set of nonlinearly coupled first-order dynamical systems to reach the *global* maximum likelihood estimate, assuming reciprocal communication links. In particular, in [8] it was shown that reaching a consensus on the state derivative, rather than on the state itself (as in [1]−[6]), allows for better resilience against coupling noise.

The consensus protocols proposed in [3]−[6] assume that the interactions among the nodes occur instantaneously, i.e., without any propagation delay. However, this assumption is not valid for large scale networks, where the distances among the nodes are large enough to introduce a nonnegligible communication delay. There are only a few recent works that studied the consensus problem for *time-continuous* dynamical systems in the presence of propagation delays [4], [9]−[12]. The discrete-time case was addressed in [1], [2, Ch. 7.3], where the authors studied alternative asynchronous linear agreement *discrete-time* algorithms. In particular, [4, 9] provided sufficient conditions for the convergence of a linear consensus protocol in the case of time-invariant *homogeneous* delays (i.e., equal delay for all the nodes) and assuming *reciprocal* communication links. The most appealing feature of the dynamical system in [4] is the convergence of the state variables to a common value, which is *known* in advance (equal to the weighted average of the initial conditions) and *delay-independent*. Unfortunately, this desired property is paid in terms of convergence capabilities, since, in the presence of homogeneous delays, the system in [4] is able to reach a consensus if and only if the delay is smaller than a given, topology-dependent, value. The protocol of [4] was generalized in [10, 12] and [11] to the case of time-invariant *nonhomogeneous* delays (but reciprocal channels) and nonreciprocal channels, respectively. The dynamical systems studied in [10, 11] are guaranteed to reach a consensus for any given set of finite propagation delays (provided that the network is strongly connected), but this common value is not related to the initial conditions of the system by a *known* function. Similar results, under weaker (sufficient) conditions on the (possibly time-varying) network topology, were obtained in [2, Ch. 7.3] for the convergence of discrete-time asynchronous agreement algorithms. This means that the final global consensus achievable by the systems in [10, 11] and [2, Ch. 7.3] is not predictable a priori, so that the protocols in the cited works cannot be used to distributively compute prescribed functions of the sensors' measurements, like decision tests or global parameter estimates.

Ideally, we would like to have a totally decentralized system that reaches a global consensus, for *any* given set of *nonhomogeneous* propagation delays (as in [10], [2, Ch. 7.3]) and *nonreciprocal* channels (as in [11], [2, Ch. 7.3]), whose final value is a *known* and *delay-independent* function of the sensors' measurements (as in [4]). In this paper we fill this lack and propose a distributed time-continuous dynamical system having all the above desired features. More specifically, we consider a set of linearly coupled first-order dynamical systems, in a network with arbitrary time-invariant topology (not necessarily strongly connected, as opposed to [9]−[12]) and nonreciprocal communication channels, modeled as a weighted directed graph with weights dependent on the physical radio channels. The links among the nodes are affected by time-invariant nonhomogeneous time offsets, taking into account the propagation delays, proportional to the relative distances among the nodes, and clock offsets among the nodes. Our main contributions are the following: i) We provide necessary and sufficient conditions ensuring local or global convergence of our dynamical system; ii) We derive the closed form expression for the consensus, as a function of the attenuation coefficient and propagation delay corresponding to each link among the sensors; iii) We show how to get a final estimate that is not biased by the network geometry and coincides with the globally optimal decision statistics that would have been computed by a centralized network having a fusion center that has ideal access to all

This work has been partially funded by the WINSOC project (Contract N0033914), and by ARL/ERO (Contract N62558-05-P-0458).

the nodes. The most appealing feature of the proposed system is the convergence of the state derivatives to a common *known* value, for *any* given set of propagation delays and nonreciprocal communication channels, with the only requirement that the network be quasi-strongly connected.

2. REACHING CONSENSUS THROUGH SELF-SYNCHRONIZATION

In this section, we first show a class of functions that can be computed with a distributed approach and then we illustrate the mechanism to carry out the computation without the need of any fusion center.

2.1. Consensus Achievable with a Decentralized Approach

If we denote by y_i, $i = 1, \ldots, N$ the (scalar) measurement taken from node i, in a network composed of N nodes, we have shown in [8] that it is possible to compute any function of the collected data expressible in the form

$$f(y_1, y_2, \ldots, y_N) = h\left[\frac{\sum_{i=1}^{N} c_i g_i(y_i)}{\sum_{i=1}^{N} c_i}\right], \qquad (1)$$

where $\{c_i\}$ are positive coefficients and $\{g_i\}$ and h are arbitrary (possibly nonlinear) real functions on $\mathbb{R}$, and i.e., $g_i, h : \mathbb{R} \mapsto \mathbb{R}$, in a totally decentralized way, i.e. without the need of a sink node. In the vector observation case, the function may be generalized to the vector form

$$\mathbf{f}(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N) = \mathbf{h}\left[\left(\sum_{i=1}^{N} \mathbf{C}_i\right)^{-1}\left(\sum_{i=1}^{N} \mathbf{C}_i \boldsymbol{g}_i(\mathbf{y}_i)\right)\right], \qquad (2)$$

where $\{\boldsymbol{g}_i\}$ and $\mathbf{h}$ are arbitrary (possibly nonlinear) real functions on $\mathbb{R}^L$, i.e., $\boldsymbol{g}_i, \mathbf{h} : \mathbb{R}^L \mapsto \mathbb{R}^L$, and $\{\mathbf{C}_i\}$ are arbitrary square positive definite matrices.

Even though the class of functions expressible in the form (1) or (2) is not the most general one, it includes many cases of practical interest, like, e.g., the computation of hypothesis testing problem, the linear ML estimation [8, 13], the sufficient statistic in detection of Gaussian processes in Gaussian noise, the maximum, the minimum, the histograms, the geometric mean of the sensors' measurements, and so on [8, 13].

2.2. How to Achieve the Consensus in a Decentralized Way

The next, most interesting question is how to achieve the aforementioned optimal statistics in a totally decentralized network. In this paper, we consider a linear interaction model among the nodes, and we generalize the approach of [8] to a network where the propagation delays are taken into account and the network is described through a weighted *directed* graph (or *digraph,* for short), which is a more appropriate model to capture the nonreciprocity of the communication links governing the interaction among the nodes.

The proposed sensor network is composed of N nodes, each equipped with four basic components: i) a *transducer* that senses the physical parameter of interest (e.g., temperature, concentration of contaminants, radiation, etc.); ii) a *local processing unit* that processes the measurement taken by the node; iii) a *dynamical system* whose state evolves according to a first-order differential equation, initialized with the local measurements, whose state evolves interactively with the states of nearby sensors; iv) a *radio interface* that transmits the state of the dynamical system and receives the state transmitted by the other nodes, thus ensuring the interaction among nearby nodes.

In the scalar observation case, the dynamical system present in node i evolves according to the following functional differential equation

$$\begin{aligned} \dot{x}_i(t) &= g_i(y_i) + \frac{K}{c_i}\sum_{j\in\mathcal{N}_i}^{N} a_{ij}\,\left(x_j(t-\tau_{ij}) - x_i(t)\right), \quad t > 0, \\ x_i(t) &= \phi_i(t), \quad t \in [-\tau,\, 0], \quad i = 1, \ldots, N, \end{aligned} \qquad (3)$$

where $x_i(t)$ is the state function associated to the i-th sensor; $g_i(y_i)$ is a function of the local observation y_i; K is a positive coefficient measuring the global coupling strength; c_i is a positive coefficient that may be adjusted to achieve the desired consensus; $\tau_{ij} = T_{ij} + d_{ij}/c$ is a delay incorporating the propagation delay due to traveling the internode distance d_{ij}, at the speed of light c, plus a possible time offset T_{ij}. The sensors are assumed to be fixed so that all the delays are constant. We also assume, realistically, that the maximum delay is bounded, with maximum value $\tau = \max_{ij} \tau_{ij}$. The coefficient a_{ij} measures the amplitude of the signal received from node i and transmitted from node j. We assume that the radio interface is such that $a_{ij} = \sqrt{P_j |h_{ij}|^2 / d_{ij}^{\eta}}$, where P_j is the power of the signal transmitted from node j; h_{ij} is a fading coefficient describing the channel between nodes i and j; η is the path loss exponent. This requires some form of channel compensation at the receiver side, if the coefficients h_{ij}'s are complex. Furthermore, we assume, realistically, that node i "hears" node j only if the power received from j exceeds a given threshold. In such a case, $a_{ij} \neq 0$, otherwise $a_{ij} = 0$. The set of nodes that sensor i hears is denoted by $\mathcal{N}_i = \{j = 1, \ldots, N : a_{ij} \neq 0\}$. Observe that, in general, $a_{ij} \neq a_{ji}$, i.e. the channels are non-reciprocal.

Because of the delays, the state evolution (3) for, let us say, $t > 0$, is uniquely defined provided that the initial state variables $x_i(t)$ are specified in the interval from $-\tau$ to 0. The initial conditions of (3) are assumed to be taken in the set of continuous bounded functions $\phi_i(t)$ mapping the interval $[-\tau,\ 0]$ to $\mathbb{R}$.

Because of the space limitation, in this paper we focus only on the case of scalar observations from the sensors. However, similar results can be generalized to the vector case [13].

2.3. Self-Synchronization

Differently from most papers dealing with average consensus problems [1]−[6] , where the global consensus was intended to be the situation where all dynamical systems reach the same *state* value, we adopt here the alternative definition already introduced in our previous work [8]. We define the network synchronization (consensus) with respect to the state *derivative*, rather than to the state.

Definition 1 *Given the dynamical system in (3), a solution* $\{\mathbf{x}_i^\star(t)\}$ *of (3) is said to be a* synchronized state *of the system, if*

$$\dot{\mathbf{x}}_i^\star(t) = \boldsymbol{\omega}^\star, \quad \forall i = 1, 2, \ldots, N. \qquad (4)$$

The system (3) is said to globally *synchronize if there exists a synchronized state as in (4), and* all *the state derivatives asymptotically converge to this common value, for* any *given set of initial conditions* $\{\boldsymbol{\phi}_i\}$, *i.e.,*

$$\lim_{t\mapsto\infty} \|\dot{\mathbf{x}}_i(t) - \boldsymbol{\omega}^\star\| = 0, \quad \forall i = 1, 2, \ldots, N, \qquad (5)$$

where $\|\cdot\|$ *denotes some vector norm and* $\{\mathbf{x}_i(t)\}$ *is a solution of (3). The synchronized state is said to be* globally asymptotically stable *if the system globally synchronizes, in the sense specified in (5). The system (3) is said to* locally *synchronize if there exist disjoint subsets of the nodes, called clusters, where the nodes in each cluster have state derivatives converging, asymptotically, to the same value, for any given set of initial conditions* $\{\boldsymbol{\phi}_i\}$.

According to Definition 1, if there exists a globally asymptotically stable synchronized state, then it must necessarily be *unique* (in the derivative). In the case of local synchronization instead, the system may have multiple synchronized clusters, each of them with a different synchronized state. In the ensuing sections, we will provide necessary and sufficient conditions for the system in (3) to locally/globally synchronize, along with the closed form expression of the synchronized state.

3. NECESSARY AND SUFFICIENT CONDITIONS FOR SELF-SYNCHRONIZATION

To derive our main results, we rely on some basic notions of digraphs theory, as briefly recalled next. A digraph $\mathscr{G}$ is defined as $\mathscr{G}=\{\mathscr{V},\mathscr{E}\}$, where $\mathscr{V}$ is the set of vertices and $\mathscr{E}\subseteq\mathscr{V}\times\mathscr{V}$ is the set of edges, with the convention that $e_{ij}=(v_i,v_j)\in\mathscr{E}$ if there exists an edge from v_j to v_i, i.e., the information flows from v_j to v_i. A digraph is weighted if a positive weight, denoted by a_{ij}, is associated with each edge e_{ij}. The out-degree of a vertex is defined as the sum of the weights of all its incoming edges. The in-degree is similarly defined. The Laplacian matrix $\mathbf{L}$ of a digraph is defined as $\mathbf{L}=\mathbf{D}-\mathbf{A}$, where $\mathbf{D}$ is the diagonal matrix of vertex out-degrees and $\mathbf{A}$ is the adjacency matrix, defined as $[A]_{ij}=a_{ij}$. A digraph is a directed tree if it has N vertices and $N-1$ edges and there exists a root vertex (i.e., a zero out-degree vertex) with directed paths to all other vertices. A directed tree is a *spanning* directed tree of a digraph $\mathscr{G}$ if it has the same vertices of $\mathscr{G}$. A forest is a collection of trees. A digraph is balanced if the out-degree of each vertices is equal to its in-degree. A digraph is *strongly* connected (SC) if any ordered pair of distinct nodes can be joined by a directed path. A digraph is *quasi-strongly* connected (QSC) if for every ordered pair of nodes v_i and v_j there exists a node r that can reach both v_i and v_j by a directed path. A digraph is *weakly* connected (WC) if any ordered pair of distinct nodes can be joined by a path, ignoring the orientation of the edges.

The next theorem is the fundamental result of this paper and it provides necessary and sufficient conditions for the proposed decentralized approach to achieve local/global consensus in the presence of propagation delays and nonreciprocal communication links.

Theorem 1 ([13]) *Let $\mathscr{G}=\{\mathscr{V},\mathscr{E}\}$ be the digraph associated to the network in (3), with Laplacian matrix $\mathbf{L}$. Let $\boldsymbol{\gamma}=[\gamma_1,\ldots,\gamma_N]^T$ be a left (normalized) eigenvector of $\mathbf{L}$ corresponding to the zero eigenvalue, i.e., $\boldsymbol{\gamma}^T\mathbf{L}=\mathbf{0}_N^T$ and $\|\boldsymbol{\gamma}\|=1$.*

Given the system in (3), assume that the following conditions are satisfied: ***a1)*** *The coupling gain K and the coefficients $\{c_i\}$ are positive;* ***a2)*** *The propagation delays $\{\tau_{ij}\}$ are finite, i.e., $\tau_{ij}\leq\tau=\max_{i\neq j}\tau_{ij}<+\infty$, $\forall i\neq j$;* ***a3)*** *The initial conditions are taken in the set of continuous bounded functions mapping the interval $[-\tau,\ 0]$ to $\mathbb{R}^N$.*

Then, system (3) globally synchronizes for any *given set of propagation delays, if and only if the digraph $\mathscr{G}$ is QSC. The synchronized state is given by*

$$\dot{x}_q^{\star}\triangleq\omega^{\star}=\frac{\sum_{i=1}^{N}\gamma_i c_i g_i(y_i)}{\sum_{i=1}^{N}\gamma_i c_i+K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\gamma_i a_{ij}\tau_{ij}},\ \forall q,\quad(6)$$

where $\gamma_i>0$ if and only if node i can reach all the other nodes of the graph through a directed path, otherwise $\gamma_i=0$.

Theorem 1 has a very broad applicability, as it does not make any particular reference to the network topology. If, conversely, the topology has a specific structure, then we may have the following forms of consensus.[1]

Corollary 1 ([13]) *Given system (3), assume that conditions* ***a1-a3*** *of Theorem 1 are satisfied. Then,*

1. *The system globally synchronizes to the state derivative*

$$\dot{x}_q^{\star}=g_r(y_r),\quad(7)$$

   $\forall q,r=1,\cdots,N$, if and only if the digraph $\mathscr{G}$ contains one spanning directed tree, with root node given by node r.

2. *The system globally synchronizes and the synchronized state is given by (6) with all γ_i's positive if and only the digraph $\mathscr{G}$ is SC. The synchronized state becomes*

$$\dot{x}_q^{\star}=\frac{\sum_{i=1}^{N}c_i g_i(y_i)}{\sum_{i=1}^{N}c_i+K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}a_{ij}\tau_{ij}},\quad(8)$$

   $\forall q=1,\cdots,N$, if and only if, in addition, the digraph $\mathscr{G}$ is balanced.

3. *The system* locally *synchronizes in K disjoint clusters $\mathscr{C}_1,\ldots,\mathscr{C}_K\subseteq\{1,\ldots,N\}$,*[2] *with synchronized state derivatives*

$$\dot{x}_q^{\star}=\frac{\sum_{i\in\mathscr{C}_k}\gamma_i c_i g_i(y_i)}{\sum_{i\in\mathscr{C}_k}\gamma_i c_i+K\sum_{i\in\mathscr{C}_k}\sum_{j\in\mathcal{N}_i}\gamma_i a_{ij}\tau_{ij}},\quad(9)$$

   $\forall q\in\mathscr{C}_k$ and $k=1,\cdots,K$, if and only if the digraph $\mathscr{G}$ is WC and it contains a forest with K strongly connected root components.

Remark 1: Robustness with respect to large propagation delays. The first important property of the proposed system resulting from Theorem 1 is its robustness against propagation delays. It turns out in fact that the convergence capability of system (3) is not affected by the propagation delays. This property represents the major difference between our system and the scheme proposed in [4, 9], where, instead, even in the special case of *homogeneous* delays (i.e., $\tau_{ij}=\tau$, $\forall i\neq j$) and *undirected* connected graph (i.e., $a_{ij}=a_{ji}$, $\forall i\neq j$), the average consensus is reached if and only if the common delay τ is smaller than a topology-dependent threshold, whose value is a decreasing function of the maximum graph degree. This implies, for example, that networks with hubs (i.e., nodes with very large degrees) that are commonly encountered in scale-free networks, are fragile against propagation delays, under the protocol of [4, 9], even in the simple case of homogeneous delays.

The reason for this difference in the convergence capabilities of the two systems in the presence of propagation delays, is a consequence of the alternative definition of global consensus that we proposed for (3), with respect to the classical one used in [1]−[6] and [9]−[11]. In fact, we do not require all the state variables to converge to a common time-independent value, but only to converge towards trajectories given by parallel straight lines. This extra flexibility provides the additional degrees of freedom that make possible the achievement of a consensus on the state derivative without requiring any constraint on the propagation delays (besides the obvious requirement of being bounded).

[1] We focus, w.l.o.g., only on WC digraphs. In the case of non WC digraphs, Corollary 1 applies to each disjoint component of the digraph.

[2] In general, the clusters $\mathscr{C}_1,\ldots,\mathscr{C}_K$ are not a partition of the set of nodes $\{1,\cdots,N\}$.

Remark 2: Effect of network topology on consensus structure. Theorem 1 generalizes all the previous (only sufficient) conditions known in the literature [2, Ch. 7.3], [4], [9]$-$ [12] for the convergence of linear agreement protocols in the presence of propagation delays. In fact, our theorem provides a full characterization of system (3) in terms of necessary and sufficient conditions for either global or local synchronization, valid for *any* possible degree of connectivity in the network (not only for SC digraphs as in [4], [9]$-$[11]), as detailed next.

In general, the digraph $\mathscr{G}$ modeling the interaction among the nodes may have one of the following structures: i) $\mathscr{G}$ contains only one directed spanning tree, with a single root node, i.e., there exists only one node that can reach all the other nodes in the network by a directed path; ii) $\mathscr{G}$ contains more than one directed spanning tree, i.e., there exist multiple nodes (possibly all the nodes), strongly connected to each other, that can reach all the other nodes by a directed path; iii) $\mathscr{G}$ is WC and contains a forest, i.e., there exists no node that can reach all the others through a directed path. In the first two cases, according to Theorem 1, system (3) achieves a *global* consensus, whereas in the third case the system forms clusters of consensus with, in general, different consensus values in each cluster, i.e., it synchronizes only *locally*. In other words, global synchronization is possible *if and only if* there exists at least one node (the root node of the spanning directed tree of the digraph) that can send its information, directly or indirectly, to all other nodes. In particular, if only one node can reach all the others, then the final consensus will depend only on the observation taken from that node (see (7)). On the other extreme, the global consensus contains contributions from *all* the nodes if and only if the graph is SC. If instead no node can reach all the others, then the information gathered in each sensor has no way to propagate through the *whole* network and thus a global consensus cannot be reached. Still, a *local* consensus is achievable among all the nodes that do influence each other (see (9)).

As an additional remark, the possibility to form clusters of consensus, rather than a global consensus, depends on the channel coefficients a_{ij}. These may be altered by changing the transmit powers P_j. According to the previous comments, the nodes with the highest transmit power will be the most influential ones. If, for example, we want to write a certain value on each node, we can use the same consensus mechanism used in this paper by assigning, for example, that value to node i, and use transmit powers such that node i is the only node that can reach every other node.

Remark 3: Closed form expression of the synchronized state. An additional important contribution of Theorem 1 is to provide a closed form expression of the synchronized state, as given in (6), valid for any network topology (not only for undirected graphs as in [4]). Expression (6) shows a dependence of the synchronized state on the network topology and propagation parameters, through the coefficients $\{a_{ij}\}$ and the delays $\{\tau_{ij}\}$.

Because of this dependence, the final consensus resulting from (6) cannot be made to coincide with the desired decision statistics as given by (1), except that in the ideal case where all the delays are equal to zero. However, expression (6) suggests also a method to get rid of the bias, as shown in the following algorithm. We let the system in (3) to evolve twice: the first time, the system evolves according to (3) and we denote by $\omega^*(\boldsymbol{y})$ the synchronized state; the second time, we set $g_i(y_i) = 1$ in (3) and we let the system evolve again, calling the final synchronized state $\omega(\mathbf{1})$. From (6), if we take the ratio $\omega^*(\boldsymbol{y})/\omega^*(\mathbf{1})$, we get

$$\frac{\omega^*(\boldsymbol{y})}{\omega^*(\mathbf{1})} = \frac{\sum_{i=1}^{N} \gamma_i c_i g_i(y_i)}{\sum_{i=1}^{N} \gamma_i c_i}, \tag{10}$$

which coincides with the ideal value achievable in the absence of delays.

Thus, using this simple two-step algorithm, we are able to control the value of the synchronized state in advance, without affecting the convergence capabilities of the system. This makes our system strongly different from the linear agreement protocols proposed in [2, Ch. 7.3], [10, 11]. In fact, the dynamical systems studied in [2, Ch. 7.3], [10, 11] are guaranteed to reach an agreement for any set of nonhomogeneous delays (provided that the digraph is QSC in [2, Ch. 7.3] and SC in [10, 11]), but this common value is not related to the sensors' measurements by a known function. In other words, the global consensus asymptotically achieved by the protocols in [2, Ch. 7.3] and [10, 11] is a priori unpredictable, so that the systems proposed in the cited works cannot be used directly to distributively compute prescribed decision tests or sufficient statistics of sensors' measurements, as given in (1).

4. NUMERICAL RESULTS AND CONCLUSIONS

In this section, we illustrate first some examples of consensus, for different network topologies. Then, we show an application of the proposed technique to an estimation problem, in the presence of random link coefficients.

Example 1: Different forms of consensus for different topologies In Figure 1, we consider two topologies (top row), namely: (a) a QSC digraph, (b) a WC (not QSC) digraph with a forest composed by two trees. For each digraph, we also sketch its decomposition into Strongly Connected Components (SCCs) (each one enclosed in a circle), and we denote by RSCC the root SCC of any spanning directed tree contained in the digraph. In the bottom row of Figure 1, we plot the dynamical evolutions of the state derivatives of system (3) versus time, for the two network topologies, together with the theoretical asymptotic values predicted by (6) (dashed line with arrows). As proved by Theorem 1, the dynamical system in Figure 1a) achieves a global consensus, since there is a set of nodes (those in the RSCC component) able to reach all other nodes. The final consensus contains only the contributions of the nodes in the RSCC, since no other node belongs to the root SCC of a spanning directed tree of the digraph. The system in Figure 1b) cannot reach a global consensus since there is no node that can reach all the others, but it does admit two disjoint clusters, corresponding to the two RSCCs, namely RSCC_1 and RSCC_2. The middle lines of Figure 1b) refer to the nodes of the SCC component, not belonging to either RSCC_1 or RSCC_2, that are affected by the consensus achieved in the two RSCC components, but that cannot affect them. Observe that, in all the cases, the state derivatives of the (global or local) clusters converge to the values predicted by the closed form expression given in (6)-(9), depending on the network topology.

Example 2: **Distributed optimal decisions through consensus**. The behaviors shown in the previous example refer to a given realization of the topology, with given link coefficients, and of the observations. In this example, we report a global parameter representing the variance obtained in the estimate of a scalar variable. Each sensor observes a variable $y_i = A_i\xi + w_i$, where w_i is additive zero mean Gaussian noise, with variance σ_i^2. The goal is to estimate ξ. The estimate is performed through the interaction system (3), with functions $g_i(y_i) = y_i/A_i$ and coefficients $c_i = A_i^2/\sigma_i^2$, chosen in order to achieve the globally optimal ML estimate. The network is composed of 40 nodes, randomly spaced over a square of size D. We set the threshold on the amplitude of the minimum useful signal so that the underlying digraph be SC. The analog system (3) is implemented in discrete time, with sampling time $T_s = 10^{-3}$ sec. The

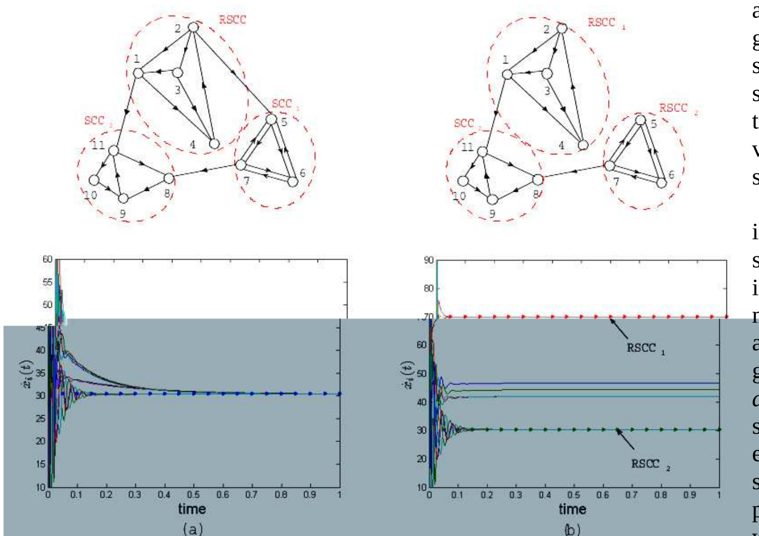

Fig. 1. Self-synchronization for two different network topologies: a) QSC digraph with three SCCs; b) WC digraph with a two trees forest; $T_s = 10^{-3}$ s, $\tau = 50T_s$, $K = 30$.

size of the square occupied by the network is chosen in order to have a maximum delay $\tau = 100T_s$. To simulate a practical scenario, the channel coefficients a_{ij} are generated as i.i.d. Rayleigh random variables, to accommodate for channel fading. Each variable a_{ij} has a variance depending on the distance d_{ij} between nodes i and j, equal to[3] $\sigma_{ij}^2 = P_j/(1 + d_{ij}^2)$.

In Figure 2, we plot the estimated average state derivative (plus and minus the estimation standard deviation), as a function of the iteration time. The averages are taken over the nodes, for 100 independent realizations of the network, where, in each realization we generated a new topology and a new set of channel coefficients and noise terms. The results refer to following cases of interest: a) ML estimate achieved with a centralized system, with no communication errors between nodes and fusion center (dotted lines); b) estimate achieved with the proposed method, with no propagation delays, as a benchmark term (dashed and dotted lines plus $\times$ marks for the average value); c) estimate achieved with the proposed method, with propagation delays (dashed lines plus stars for the average value); d) estimate achieved with the two-step estimation method leading to (10) (solid lines plus circles for the average value). From Figure 2,

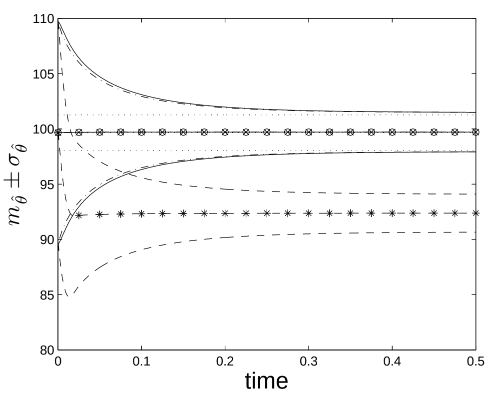

Fig. 2. Estimated parameter vs. convergence time.

we can see that, in the absence of delays, the (decentralized) iterative algorithm based on (3) behaves, asymptotically, as the (centralized) globally optimal ML estimator. In the presence of delays, we observe a clear bias (dashed lines), due to the large delay values, but still with a final estimation variance close to the ML estimator's. Interestingly, if the two-step procedure leading to (10) provides results very close to the optimal ML estimator, with no apparent bias, in spite of the large delays and the random channel fading coefficients.

In conclusion, in this paper we have proposed a totally decentralized sensor network scheme capable to reach globally optimal decision tests through local exchange of information among the nodes, in the presence of nonreciprocal communication channels and inhomogeneous time-invariant propagation delays. Differently from the average consensus protocols available in the literature, our system globally synchronizes for *any* set of (finite) propagation delays *if and only if* the underlying digraph is QSC, with a final synchronized state that is a *known* function of the sensor measurements. In general, the synchronized state depends on the propagation parameters, such as delays and the communication channels. Nevertheless, exploiting our closed form expression for the final consensus values, we have shown how to recover an unbiased estimate, for any realization of delays and channel coefficients, without the need to know or estimate these coefficients. If we couple the nice properties mentioned above with the properties reported in [8], where we showed that, in the absence of delays, the consensus protocol proposed in this paper and in [8] is also robust against coupling noise, we have, overall, a good candidate for a distributed sensor network.

[3] We use the attenuation factor $1/(1 + d_{ij}^2)$ instead of $1/d_{ij}^2$ to avoid the undesired event that, for $d_{ij} < 1$ the received power might be greater than the transmitted power.

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